Testable Implications of General Equilibrium Models: An Integer Programming Approach

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"To apply the methodology to large data sets, it is necessary to devise a computationally efficient algorithm for solving large families of equilibrium inequalities."

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Sonnenschein-Mantel-Debreu

- Consumers with utility functions u^j, endowments ε^j choose consumption given the prices p^r.

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- *"Anything goes!"* (Mas-Colell, Whinston and Green, 1995)

The Equilibrium Manifold

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- Brown-Matzkin (1996): What are the necessary and sufficient conditions on a finite data set on (equilibrium) prices, aggregate endowments and individual incomes such that this data is consistent with observations from the equilibrium manifold.
- Using Revealed preference theory, they show that not every data set is consistent with equilibrium behavior.

Proof outline

1. Develop Revealed Preference conditions that guarantee the existence of utility functions and individual consumption bundles such that:
    - Individual expenditure equals individual income,
    - Individual consumption sums to aggregate endowment,
    - Individual consumption maximizes the individual utility function given the available income.

   These conditions form a set of polynomial inequalities.

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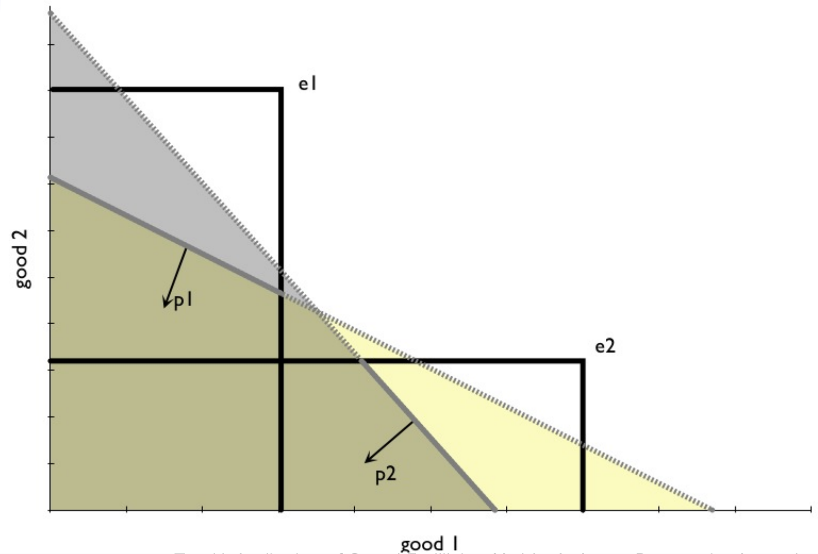
2. Employ Tarski-Seidenberg algorithm: Any finite system of polynomial inequalities can be reduced to an equivalent finite family of polynomial inequalities in the coefficients of the given system.

3. Provide a counterexample.

Counterexample



Other research

- Public goods (Snyder, 1999); financial markets (Kübler, 2003); random preferences (Carvajal, 2004); Pareto efficiency (Bachman, 2006); interdependent preferences (Deb, 2009); externalities (Carvajal, 2009),. . .

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  2. Use the Tarski-Seidenberg algorithm.
  3. Provide a counterexample.
- Tarski-Seidenberg: can it be used to operationalize the general equilibrium conditions?

Tarski-Seidenberg

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". . . to apply the method to large data sets, researchers would need an efficient way to solve large systems of nonlinear polynomial inequalities." *(Rizvi, 2006)*

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  - Alternative algorithms:
    Brown and Kannan (2008): Enumerate preference orderings.
    Brown and Kannan (2008): VC algorithm.
- We illustrate the IP algorithm using US data.

Testable implications on the equilibrium manifold

Definition (General equilibrium rationalizability)

$\{p_t, \varepsilon_t, I_t^j\}_{t\in T}$ is general equilibrium rationalizable if there exist x_t^j and $u^j(.)$ such that:

- $\sum_j x_t^j = \varepsilon_t$,
- $p_t' x_t^j = I_t^j$,
- $x_t^j \in \arg\max_x \ u^j(x) \ \text{ s.t. } \ p_t' x \leq I_t^j$.

Individual rationalizability

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$\{p_t, x_t\}$ is individual rationalizable if there exist a utility function $u(.)$ such that:

- $x_t \in \arg\max_x \; u(x) \;$ s.t. $\; p_t' x \leq I_t$.

Individual rationalizability

Theorem (Afriat, 1967), (Varian, 1982)

A data set $\{p_t, x_t\}_{t \in T}$ is individual rationalizable if :

- There exist numbers U_t and $\lambda_t > 0$ such that: $U_t - U_v \leq \lambda_v p_v'(x_t - x_v)$,
- $\{p_t, x_t\}_{t \in T}$ satisfies GARP.

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Characterization

Theorem [Brown and Matzkin, 1996]

$\{p_t, \varepsilon_t, I_t^j\}_{t \in T}$ is general equilibrium rationalizable if there exist x_t^j such that:

- $\sum_j x_t^j = \varepsilon_t$,
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    - or $\{p_t, x_t^j\}_{t \in T}$ satisfies GARP.

Computational complexity

$$\sum_j x_t^j = \varepsilon$$
$$p_t' x_t^j = I_t^j$$
$$U_t^j - U_v^j \leq \lambda_v^j p_v' (x_t^j - x_v^j)$$

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$\sum_j x_t^j = \varepsilon$	linear
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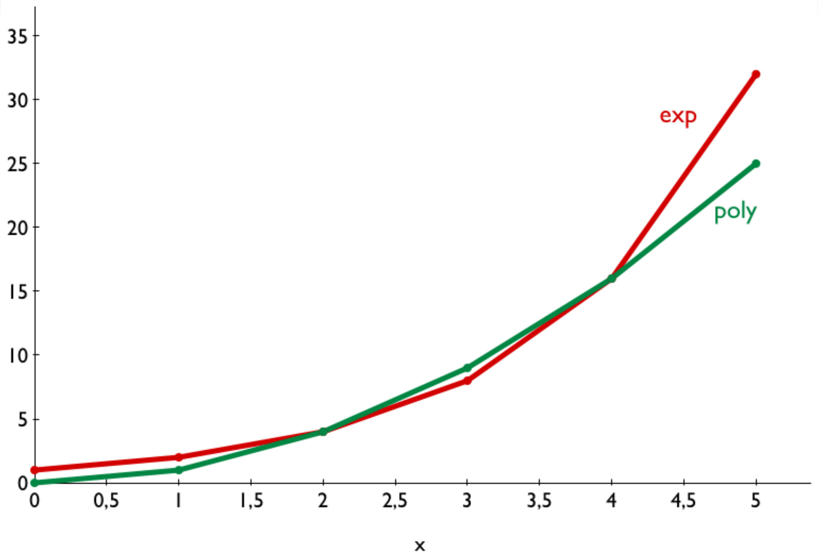
- Polynomial time: efficient, (e.g. x^2).
- Exponential time: inefficient (e.g. 2^x).
- NP-complete: no efficient algorithm exists (unless $P = NP$).

Exponential versus polynomial



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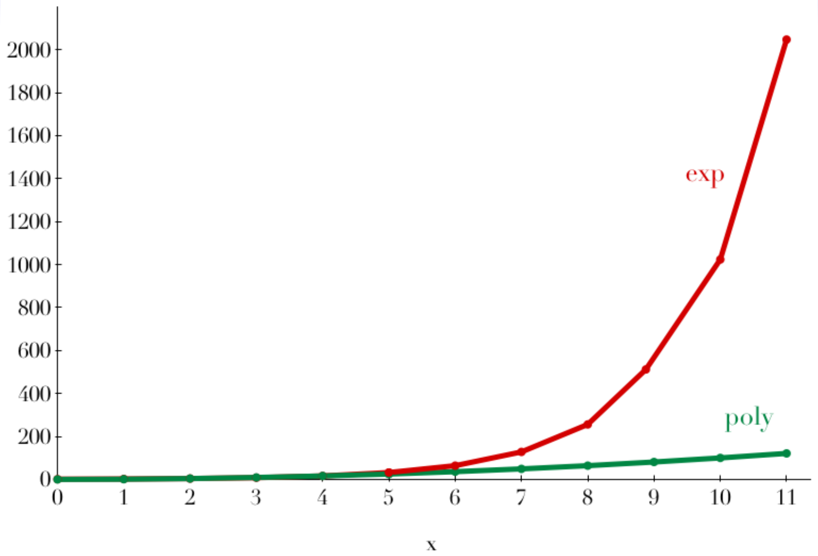

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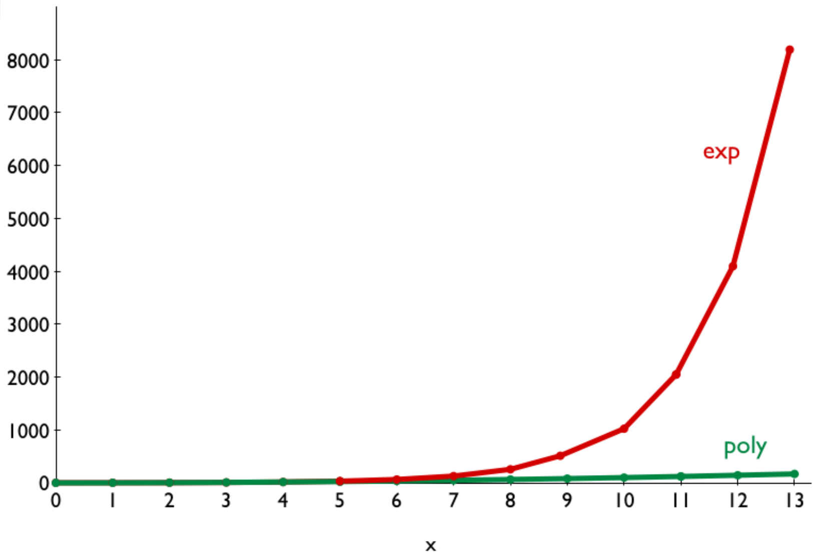

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- It is also very flexible in order to analyze alternative general equilibrium models.

The IP approach to GARP

Set $r(v, t) = 1$ if and only if $x_t R x_v$

GARP conditions

$$\text{I: if} \quad p_t' x_t \geq p_t' x_v \quad \text{then} \quad x_t R x_v,$$

$$\text{II: if} \quad x_t R x_v R x_k \quad \text{then} \quad x_t R x_k,$$

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III: if $x_t R x_v$ then $p_v' x_v \leq p_v' x_t$,

$$(1 - r(t,v))A \geq p_v'(x_v - x_t)$$

The IP program

Exchange Economy

$\{p_t, \varepsilon_t, I_t^j\}_{t \in T}$ is general equilibrium rationalizable if there exist x_t^j such that:

- $\sum_j x_t^j = \varepsilon_t$,
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- $\{p_t, x_t^j\}_{t \in T}$ satisfies GARP.

The IP program

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$\{p_t, \varepsilon_t, I_t^j\}_{t\in T}$ is general equilibrium rationalizable if there exist x_t^j and $r^j(t,v) \in \{0,1\}$ such that:

- $\sum_j x_t^j = \varepsilon_t$,
- $p_t' x_t^j = I_t^j$,
- $p_t'(x_t^j - x_v^j) < r^j(t,v)A$,
- $r^j(t,v) + r^j(v,k) \leq 1 + r^j(t,k)$,
- $(1 - r^j(t,v))A \geq p_v'(x_v^j - x_t^j)$.

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Lower bound on income

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Assignable information on consumption

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- $(1 - r^j(t,v))A \geq p_v'(x_v^j - x_t^j)$.
- $x_t^j \geq \bar{x}_t^j$

The IP program

Pareto provision of public goods

$\{p_t, \varepsilon_t, I_t^j\}_{t \in T}$ is general equilibrium rationalizable if there exist x_t^j, P_t^j and $r^j(t, v)$ such that:

- $\sum_j x_t^j = \varepsilon_t$,
- $p_t' x_t^j + P_t^{j\prime} Q_t = I_t^j$,
- $p_t'(x_t^j - x_v^j) + P_t^{j\prime}(Q_t - Q_v) < r^j(t, v) I_t^j$,
- $r^j(t, v) + r^j(v, k) \leq 1 + r^j(t, k)$,
- $(1 - r^j(t, v)) I_v^j \geq p_v'(x_v^j - x_t^j) + P_v^{j\prime}(Q_v - Q_t)$.
- $\sum_j P_t^j = P_t$.

The IP program

Private provision of public goods

$\{p_t, \varepsilon_t, I_t^j\}_{t \in T}$ is general equilibrium rationalizable if there exist x_t^j, P_t^j and $r^j(t, v)$ such that:

- $\sum_j x_t^j = \varepsilon_t$,
- $p_t' x_t^j + P_t^{j\prime} Q_t = I_t^j$,
- $p_t'(x_t^j - x_v^j) + P_t^{j\prime}(Q_t - Q_v) < r^j(t, v) I_t^j$,
- $r^j(t, v) + r^j(v, k) \leq 1 + r^j(t, k)$,
- $(1 - r^j(t, v)) I_v^j \geq p_v'(x_v^j - x_t^j) + P_v^{j\prime}(Q_v - Q_t)$.
- $\max_j\{P_t^j\} = P_t$.

Application

- US aggregate data.
- T: 12 observations: 1997-2008
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- T: 12 observations: 1997-2008
- ε_t: 18 goods
- I_t^j: national incomes for 51 states or 8 regions.
- IP test for 51 states: pass after 19 minutes.
- We choose the 8 regions for power analysis.

Power analysis

- Assignable information: $\bar{x}_t^j = \dfrac{I_t^i}{p_t \varepsilon_t} \varepsilon_t$.
- Require that $x_t^j \geq \kappa \bar{x}_t^j$. ($\kappa \in [0, 1]$)
- The lower κ, the less assignable information.
- Alternative scenario: defense is a public good.

Power Results

Test	Power for basic model	Power for model with public consumption
$\kappa = 1.0$	0.6372	0.5934
$\kappa = 0.9$	0.2946	0.0479
$\kappa = 0.8$	0.1211	0.0084
$\kappa = 0.7$	0.0544	0.0034
$\kappa = 0.6$	0.0333	0.0016
$\kappa = 0.5$	0.0180	0.0008
$\kappa = 0.4$	0.0135	0.0003
$\kappa = 0.3$	0.0088	0.0001
$\kappa = 0.2$	0.0064	0.0001
$\kappa = 0.1$	0.0032	0.0001
$\kappa = 0.0$	0.0000	0.0000

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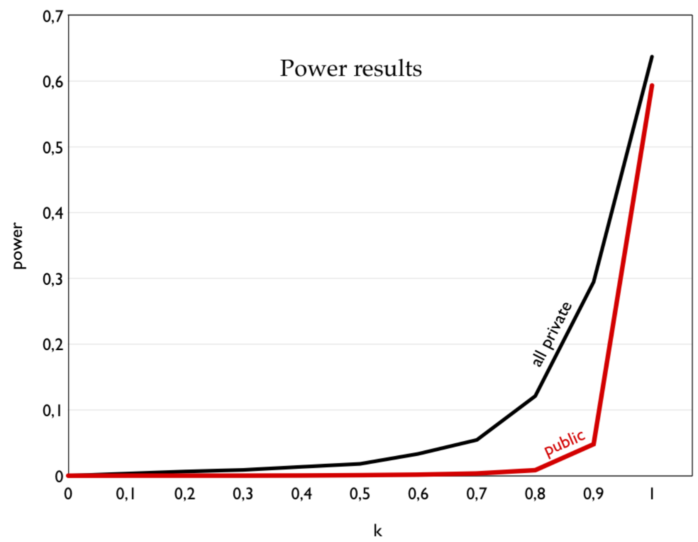

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- MIP allows easily implementable necessary and sufficient conditions.
- The approach is flexible to consider extension towards other gen. eq. models (e.g. with public goods).
- Assignable info is important to increase power.

Conclusion

- Testable restrictions on equilibrium manifold are difficult to verify: NP-Complete.
- MIP allows easily implementable necessary and sufficient conditions.
- The approach is flexible to consider extension towards other gen. eq. models (e.g. with public goods).
- Assignable info is important to increase power.
- Future topics:
  - heuristics,
  - special cases that are efficiently verifiable (e.g. quasi-linear utility)
  - recovery,
  - goodness of fit.